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SIGHT DISTANCE AT UNDERCROSSINGS

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SIGHT DISTANCE AT UNDERCROSSINGS

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SYNOPSIS

Daylight sight distances on highway sag vertical curves may be limited by a structure crossing the right-of-way. Conventional formulas for the determination of this sight distance are derived for the special case when the critical edge of the structure is directly above the vertex of the vertical curve. Despite this limitation, the formulas are considered to be applicable regardless of the position of the critical edge with respect to the vertex of the vertical curve.

This paper derives formulas which are correct for any position of the structure.

INTRODUCTION

In Fig. 1, 2, and 3, A is the critical edge of the underpass structure. $\overline{MN}$ is the line of sight with M the driver's eye at a height h above the pavement and N a point on the object the driver must see at a height h_1 above the pavement. The horizontal projection of $\overline{MN}$ is the sight distance, S. The grades of the entering and leaving tangents are g_1 and g_2 respectively with direction of travel always assumed to be from left to right unless otherwise noted. The length of the vertical curve is L and a is a constant of the parabola equal to $\frac{g_2 - g_1}{2L}$. All horizontal distances are expressed in stations and vertical distances in feet. Grades are expressed in percent.

Case I—Sight Distance Less Than Length of Vertical Curve. In Fig. 1, A is any point such that the line of sight remains entirely within the vertical curve. The vertical clearance is $\overline{AC}$ and the horizontal distance from A to the object end of the line of sight is kS where k is an unknown ratio. $\overline{FJ}$ is a tangent of the parabola drawn parallel to chord $\overline{HK}$. The point of tangency, E, will be at the midpoint of $\overline{FJ}$. The clearance, $\overline{AC}$, is equal to $\overline{AB} + \overline{BD} - \overline{CD}$. By proportion

$$\overline{AB} = h_1 + (h - h_1)\,k$$

From the rule of offsets from a tangent to a parabola

$$\overline{BD} = \overline{HF} = \frac{1}{4}\,a S^2$$

and

$$CD = a\left(kS - \frac{S}{2}\right)^2$$

Letting the clearance $\overline{AC}$ equal c and substituting the above expressions for its component parts

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$$c = h_1 + (h-h_1)\,k + \frac{1}{4}\,a\,S^2 - a\,(kS - \frac{S}{2})^2$$

Simplifying

$$a\,k^2S^2 - a\,kS^2 - (h-h_1)\,k = h_1 - c \tag{1}$$

Differentiating with respect to k and equating $\frac{dS}{dk}$ to zero, yields the value of k for which S is a minimum,

$$k = \frac{1}{2} + \frac{h - h_1}{2\,a\,S^2}$$

Substituting this value of k in Equation 1 and simplifying

$$S^2 = \frac{2c - (h + h_1)}{a} \pm \frac{2}{a}\sqrt{(c-h)\,(c-h_1)} \tag{2}$$

The values recommended by the AASHO[2] are:

$h = 6$ feet

$h_1 = 1.5$ feet

$c = 14$ feet

Substituting these values in Equation 2

$$S^2 = \frac{40.5}{a} \tag{3}$$

Substituting $\frac{g_2 - g_1}{2L}$ for a and solving for L

$$L = \frac{g_2 - g_1}{81}\,S^2 \tag{4}$$

The AASHO formula is

$$L = \frac{g_2 - g_1}{82}\,S^2 \tag{5}$$

Case II—Sight Distance Greater Than Length of Vertical Curve. In Fig. 2, A is the critical edge of the obstruction, distant d from the vertex of the vertical curve. d is assumed positive to the right and it must be less than L/2. The distance from A to the object end of the line of sight is kS where k is an unknown ratio.

The elevations of M,A, and N, above the vertex, are

$$\text{El. } M = -g_1\,(S - kS - d) + h$$

$$\text{El. } A = g_2 d + a\,(\frac{L}{2} - d)^2 + c$$

$$\text{El. } N = g_2\,(kS + d) + h_1$$

By proportion

$$\frac{\text{El. } M - \text{El. } N}{S} = \frac{\text{El. } A - \text{El. } N}{kS}$$

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2. A Policy on Grade Separations for Intersecting Highways—American Association of State Highway Officials, 1944, p. 28.

or

$$k\left[-g_1(S-kS-d)+h-g_2(kS+d)-h_1\right]=g_2d+a\left(\frac{L}{2}-d\right)^2+c-g_2(kS+d)-h_1 \quad (6)$$

Simplifying and substituting $\frac{g_2-g_1}{2L}$ for a

$$kS-k^2S-kd+\frac{h-h_1}{g_2-g_1}k=\frac{L}{8}-\frac{d}{2}+\frac{d^2}{2L}+\frac{c-h_1}{g_2-g_1} \quad (7)$$

Differentiating S with respect to k, and setting $\frac{dS}{dk}$ equal to zero

$$S-2kS-d+\frac{h-h_1}{g_2-g_1}=0 \quad (8)$$

For a minimum value of S

$$k=\frac{1}{2}-\frac{d}{2S}+\frac{1}{2S}\,\frac{h-h_1}{g_2-g_1} \quad (9)$$

Solving Eq. 7 for L

$$L^2-8L\left[kS(1-k)-d\left(k-\frac{1}{2}\right)+k\frac{h-h_1}{g_2-g_1}-\frac{c-h_1}{g_2-g_1}\right]+4d^2=0 \quad (10)$$

Calling the expression in the bracket F

$$L=4F+2\sqrt{4F^2-d^2} \quad (11)$$

Substituting the value of k from Eq. 9 in the expression for F and simplifying

$$F=\frac{S}{4}+\frac{1}{4S}\left[\frac{h-h_1}{g_2-g_1}-d\right]^2-\frac{2c-h-h_1}{2(g_2-g_1)} \quad (12)$$

With $h = 6'$, $h_1 = 1.5'$ and $c = 14'$ as before

$$F=\frac{S}{4}+\frac{1}{S}\left[\frac{2.25}{g_2-g_1}-\frac{d}{2}\right]^2-\frac{10.25}{g_2-g_1} \quad (13)$$

Notice that Eq. 13 assumes that the clearance at the critical edge is 14 feet. This is not necessarily so as the critical edge may be the one with a clearance of more than 14 feet. To determine the critical edge, it is necessary to know whether the line of sight has a positive or negative slope in the direction of travel of the vehicle. To avoid confusion, the vehicle will always be assumed to be traveling from left to right unless otherwise noted.

If the elevation of N minus the elevation of M is called E, the sign of E will be the same as that of the slope of the line of sight. Then

$$E=g_2(kS+d)+h_1+g_1(S-kS-d)-h \quad (14)$$

Simplifying and substituting for k its value in Eq. 9, with $h = 6'$ and $h_1 = 1.5'$ as before

$$E=g_1S-2.25+\frac{S+d}{2}(g_2-g_1) \quad (15)$$

Should Eq. 15 indicate that the slope of the line of sight is such that the critical edge is the one with a clearance of more than 14', it is necessary to modify Eq. 13 by adding the additional clearance to the constant of the last term.

To illustrate the use of Eq. 11, 13 and 15, the following problem will be solved:

A minus 2 percent slope meets a plus 3 percent slope at a sag at Sta. 10 + 00 El. 100. An overcrossing structure 100 feet wide which crosses the right-of-way at right angles, extends from Sta. 7 to Sta. 8. The required sight distance is 1320 feet. What length of vertical curve is required?

Since it is not at once apparent which is the critical edge, assume the critical edge to be at Sta. 7, i.e., d = -3. Then solving Eq. 15, E is found to be -3.15. The magnitude is not important; the sign is important as it confirms the assumption as to the critical edge. Solving Eq. 13, F is found to be 1.538 and from Eq. 11, L is 7.51 sta.

This is not a complete solution as the sight distance in the opposite direction must be considered. Solving Eq. 15 with d = +3, g_1 = -3, and g_2 = +2, E is negative and the critical edge is that at Sta. 8. The clearance at Sta. 8 must be greater than 14 feet, the clearance at Sta. 7. To determine this additional clearance it is necessary to assume a value of L. Assuming L = 7.50 sta., the clearance at Sta. 8 corresponding to a 14-foot clearance at Sta. 7, is 15.17. This changes Eq. 13 to

$$F = \frac{S}{4} + \frac{1}{S}\left[\frac{2.25}{g_2-g_1} - \frac{d}{2}\right]^2 - \frac{11.42}{g_2-g_1} \qquad (16)$$

and F is 1.081 and L is 5.97.

Hence for a sight distance in both directions of 1320 feet, the vertical curve must be 750 feet long. The AASHO formula for L when S > L is

$$L = 2S - \frac{82}{g_2-g_1} \qquad (17)$$

This formula requires a vertical curve for these conditions to be 1000 feet long as compared with the correct value of 750 feet. Shortening the curve to 750 feet reduces the required elevation of the critical edge by 0.81 feet.

A comparison of the value of L obtained from Eq. 11 and Eq. 17 for various values of S, g_1, g_2, and d is made in Table 1. In each case the obstruction has been so located with respect to the vertex that the critical edge is the one with the minimum (14-foot) clearance. The length of curve required by Eq. 11 has been computed for travel in both directions. Where the tabular value of d is, for example, +1, the obstruction is one station to the right of the vertex. In computing the value of L for travel from right to left the value of d used is -1. The grade to the left of the vertex is always numerically equal to g_1 but, of course, its sign is changed when travel from right to left is considered.

Referring again to the problem, if the overcrossing structure were shifted 50 feet further away from the vertex, the value of L determined by Eq. 11 would be imaginary. A glance at Table 1 shows that, for any given set of conditions, there is a critical value of d beyond which Eq. 11 is not applicable, i.e., L becomes imaginary. This critical value of d is 2F.

When d exceeds 2F another relationship is required. This relationship will now be considered as Case IIa. It is also applicable for values of d less than the critical but it is not as convenient to use as Eq. 11.

Case II a—Sight Distance Greater Than Length of Vertical Curve. In Fig. 3, A is any point within the sag. The line of Sight, $\overline{MN}$, has an unknown slope, G. $\overline{MO}$ and $\overline{ON}$ are parallel to the grade tangents and vertically above them h and h_1, respectively. The coordinates of A with respect to coordinate axes through 0 are x and y. The coordinates of the Vertex, V, with respect to the same axes are w and v. Point A divides the sight distance, S into segments, d_1 and d_2.

$$y = -g_1(d_1 - x) + Gd_1$$

and

$$y = g_2(d_2 + x) - Gd_2$$

Solving these expressions for d_1 and d_2 and adding

$$S = \frac{y-g_1x}{G-g_1} + \frac{y-g_2x}{g_2-G} \qquad (18)$$

Differentiating this expression with respect to G and equating $\frac{dS}{dG}$ to zero,

$$\frac{y-g_1x}{(G-g_1)^2} = \frac{y-g_2x}{(g_2-G)^2} \qquad (19)$$

Solving for G

$$G = (y/x) \pm \sqrt{(y/x)^2 - (y/x)(g_1+g_2) + g_1g_2} \qquad (20)$$

The value of G obtained by substituting the known values of x, y, g_1 and g_2 in Eq. 20, is then used in Eq. 18 which may be rearranged to

$$S = \frac{(y-xG)(g_2-g_1)}{(G-g_1)(g_2-G)} \qquad (21)$$

The coordinates of the vertex are, with proper regard to sign,

$$w = \frac{h-h_1}{g_1-g_2} \qquad (22)$$

and

$$v = \frac{g_1(h-h_1)}{g_1-g_2} - h \qquad (23)$$

To illustrate the use of Eq. 20, 21, 22 and 23, assume the data given in the problem except that the overcrossing extends from Sta. 6 + 50 to Sta. 7 + 50. From the previous solution it is logical to assume that the curve will be shorter than 700 feet. The elevation of the critical edge is then 121.00 and the distance from the vertex is -3.50 sta. From Eq. 22 and 23, w and v are -0.90 sta. and -4.20 feet respectively. Then x and y are -4.40 sta. and 16.80 feet respectively. From Eq. 20, G is -0.30 percent and from Eq. 21, S is 13.80 sta. Thus the actual sight distance is greater than the required sight distance without a vertical curve. If S had been less than 1320 feet it would have been necessary to increase y by introducing a vertical curve longer than 700 feet or increasing the clearance. A check of the sight distance in the opposite direction shows that the critical edge is the one at Sta. 7 +50 and S is 14.63 sta.

So far as daylight sight distance is concerned, it is not necessary to have a vertical curve in this sag. The length of the vertical curve would be determined by some other design criterion.

CONCLUSIONS AND COMMENTS

For Case I, sight distance less than length of vertical curve, the conventional formula is, for all practical purposes, identical with the correct formula. This will be true for any reasonable values of c, h, and h_1.

For Case II the conventional formula (Eq. 17) is remarkably accurate if the critical edge is within about 200 feet of the vertex of the vertical curve, and if sight distance is considered in each direction (Since passing sight distances

are involved in most Case II practical problems, both directions must be considered). However when the critical edge is more than 200 feet from the vertex, the conventional formula will usually yield results which are considerably in error.

As the critical edge moves further away from the vertex, Eq. 11 becomes invalid and it is necessary to resort to Eq. 21 to determine the sight distance. Both of these equations when used in conjunction with the equations from which they were developed, permit exact location of the line of sight. This permits rigorous solutions for complicated sight distance problems such as a sag with more than one structure crossing the right-of-way.

S	g_1	g_2	d	Equation 11 $\overrightarrow{L}$	Equation 11 $\overleftarrow{L}$	Minimum L Eq. 11	Minimum L Eq. 17
13.2	-2	+3	0	10.12	10.12	10.12	
			+1	9.58	10.15	10.15	
			+2	8.23	9.62	9.62	10
			+3	N.A.	7.51	7.51	
16.25	-3	+1	0	12.15	12.15	12.15	
			-1	12.24	11.66	12.24	
			-2	11.84	10.59	11.84	12
			-3	10.73	7.87	10.73	
			-3.5	9.41	N.A.	9.41	
18	-1.5	+2	0	12.75	12.75	12.75	
			+1	12.24	12.83	12.83	
			+2	11.17	12.48	12.48	12.60
			+3	8.79	11.46	11.46	
			+3.5	N.A.	10.38	10.38	
21	-1	+2	0	14.88	14.88	14.88	
			+1	14.41	15.01	15.01	
			+2	13.50	14.76	14.76	14.67
			+3	11.84	14.03	14.03	
			+4	N.A.	12.40	12.40	

TABLE I

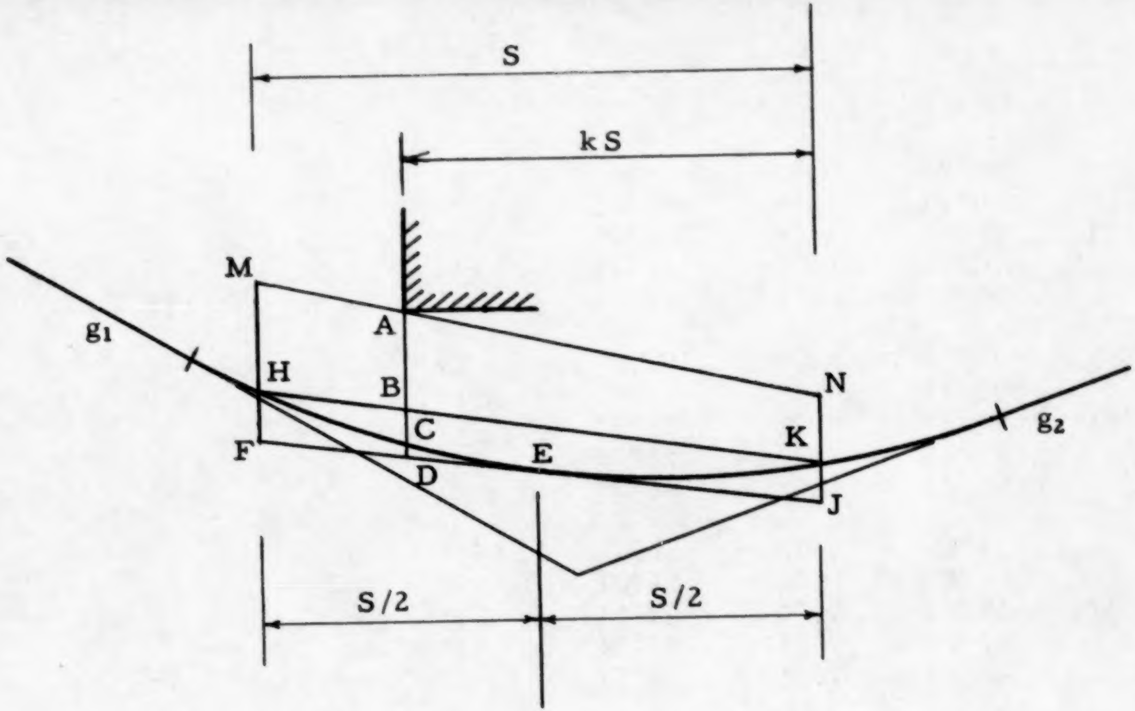

Fig. 1

Sight Distance Less Than Length of Vertical Curve

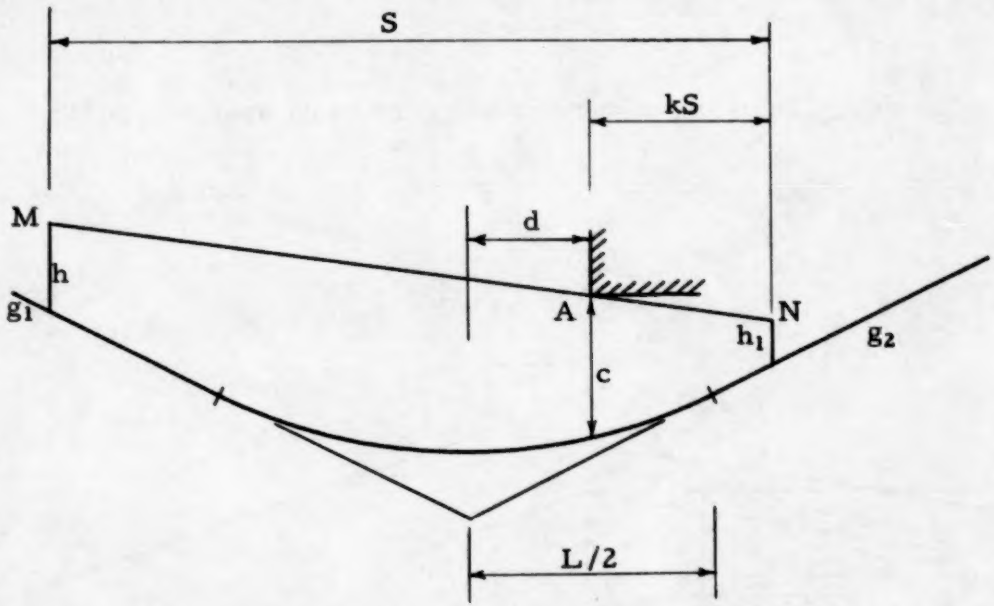

Fig. 2

Sight Distance Greater Than Length of Vertical Curve

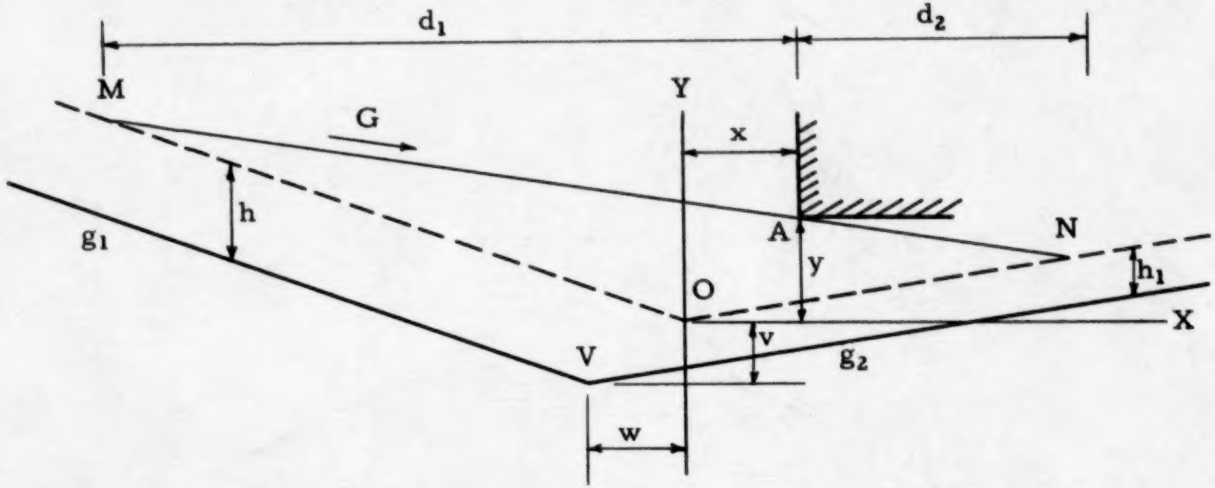

Fig. 3

Sight Distance Greater Than Length of Vertical Curve